Quantitative Economics for the Evaluation of the European Policy

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Capital mobility

Capital reallocation across EU regions seems to face less difficulties, therefore it appears as a good candidate to eliminate regional disparities: **we will see below that this is not true**.

As for labour, assume that capital is paid to its marginal productivity

$$r_R = \frac{\partial Y_R}{\partial K_R} = \alpha K_R^{\alpha-1} \left(A_R L_R\right)^{1-\alpha} = \alpha \frac{Y_R}{K_R} \tag{1}$$

and

$$r_P = \frac{\partial Y_P}{\partial L_P} = \alpha K_P^{\alpha-1} \left(A_P L_P\right)^{1-\alpha} = \alpha \frac{Y_P}{K_P} \tag{2}$$

and it is reallocated according to differences in the return, i.e.

$$\dot{K}_R > 0 \text{ if } r_R > r_P, \tag{3}$$

and analogously

$$\dot{K}_P > 0 \text{ if } r_P > r_R. \tag{4}$$

Capital mobility (cont.d)

In equilibrium returns on capital should be the same in the two regions, therefore:

$$r_R^E = r_P^E \Rightarrow \frac{Y_R}{K_R} = \frac{Y_P}{K_P} \Rightarrow \frac{Y_R}{Y_P} = \frac{K_R}{K_P} \Rightarrow \frac{Y_R/L_R}{Y_P/L_P} = \frac{K_R/L_R}{K_P/L_P} \qquad (5)$$

This means that the ration between GDP per worker of two regions fully reflect the ratio of capital per worker of the two regions.
But since in equilibrium $r_R^E = r_P^E$, then:

$$\alpha K_R^{\alpha-1} (A_R L_R)^{1-\alpha} = \alpha K_P^{\alpha-1} (A_P L_P)^{1-\alpha} \qquad (6)$$

i.e.

$$\left(\frac{K_R}{L_R}\right)^{\alpha-1} A_R^{1-\alpha} = \left(\frac{K_P}{L_P}\right)^{\alpha-1} A_P^{1-\alpha} \qquad (7)$$

and finally:

$$\frac{K_R/L_R}{K_P/L_P} = \frac{A_R}{A_P} \qquad (8)$$

Capital mobility (cont.d)

Therefore the free reallocation of capital lead to regional disparities reflecting the different level of technological progress, i.e.

$$\frac{Y_R/L_R}{Y_P/L_P} = \frac{A_R}{A_P}. \tag{9}$$

In general, A can reflect a different level of human capital, a different level of knowledge, or a different level of total facto productivity. In any case **no convergence happens in equilibrium**.

If A has also some spatial dependence, as in the model discussed in the previous classes, then we have also a potential explanation of the observed geographical clusters of regions with high and low GDP per worker.

From theory to the econometric model

The effect of a continue reallocation of capital among regions could be represented as:

$$\log r_{i,t} = (1-\phi)\log r_{i,t-\tau} + \phi \log \bar{r} + \epsilon_{i,t}, \tag{10}$$

where $r_{i,t}$ is the **real interest rate** of region i at year t, τ is **lag** in the dynamic of interest rate, ϕ is the parameter measuring the "**frictions**" in the reallocation of capital among regions, $\bar{r}$ is the **equilibrium interest rate**, and $\epsilon_{i,t}$ is a i.i.d. random shock.

- Eq. (10) states that $r_{i,t}$ follows a autoregressive process, and in equilibrium we should observe all regions with an interest rate equal to $\bar{r}$ unless random deviations. This is like ase the log-linearization of the dynamics of income around its equilibrium.
- For a small open economy $\bar{r}$ reprensents the international real interest rate. This can vary over time.
- In general $\epsilon_{i,t}$ could have both a full variance-covariance matrix and autoccorelated, i.e. $E\left[\epsilon_{i,t}, \epsilon_{j,t}\right] \neq 0$ and $E\left[\epsilon_{i,t}, \epsilon_{i,t'}\right] \neq 0$.

From theory to the econometric model (cont.d)

Since

$$r_{i,t} = \alpha Y_{i,t}/K_{i,t},$$

and

$$K_{i,t} = \left[\frac{Y_{i,t}}{(A_{i,t}L_{i,t})^{1-\alpha}}\right]^{1/\alpha}$$

we have:

$$r_{i,t} = \alpha \left(\frac{A_{i,t}L_{i,t}}{Y_{i,t}}\right)^{(1-\alpha)/\alpha} = \alpha \left(\frac{A_{i,t}}{y_{i,t}}\right)^{(1-\alpha)/\alpha},$$

where $y_{i,t} \equiv Y_{i,t}/L_{i,t}$ is the GDP per worker of region i.

Substituting in Eq. (10):

$$\log \alpha \left(\frac{A_{i,t}}{y_{i,t}}\right)^{(1-\alpha)/\alpha} = (1-\phi)\log \alpha \left(\frac{A_{i,t-\tau}}{y_{i,t-\tau}}\right)^{(1-\alpha)/\alpha} + \phi \log \bar{r} + \epsilon_{i,t},$$

i.e.

$$\log\left(\frac{y_{i,t}}{y_{i,t-\tau}}\right) = \left(\frac{\phi\alpha}{1-\alpha}\right)\log\left(\frac{\alpha}{\bar{r}}\right) - \phi\log y_{i,t-\tau} + \tag{11}$$

$$+ \log\left(\frac{A_{i,t}}{A_{i,t-\tau}}\right) + \phi\log A_{i,t-\tau} + \epsilon_{i,t}. \tag{12}$$

Taking the usual approximation that $\log x_t/x_{t-\tau} \approx g_x\tau$, i.e. that the logarithm of the ratio of x at the end and at the begin of the period $[t-\tau, t]$ is approximately equal to the average growth rate times the number of years τ of the period we have:

$$g_{y_i} \approx \left[\frac{\phi\alpha}{\tau(1-\alpha)}\right]\log\left(\frac{\alpha}{\bar{r}}\right) - \left(\frac{\phi}{\tau}\right)\log y_{i,t-\tau} + g_{A_i} + \left(\frac{\phi}{\tau}\right)\log A_{i,t-\tau} + \frac{\epsilon_{i,t}}{\tau}. \tag{13}$$

• Assuming that the average growth rate of technological progress of region i, g_{A_i}, depends from its neighbours, and initial level of technological progress, $A_{i,t-\tau}$, also depend on the initial levels of technological progress of neighbours, we have that Eq. (13) suggests to estimate a **fixed-effect spatial panel model**.

• Differences in the equilibrium level of GDP per worker among regions will be due to different initial levels of technology but also to different initial levels of technology of my neighbours.

• $A_{i,t-\tau}$ should be taken as any initial condition/variable which affect the total factor productivity, as norms, geography, social capital, etc.