

Quantitative Economics for the Evaluation of the European Policy

Dipartimento di Economia e Management

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- Solow model with poverty trap or better **multiple equilibria** (but why only two?)
 - ☑ **endogenous investment rate**
 - ⇨ **endogenous growth rate of population/employment**
 - **increasing returns to scale (change in output composition)**
 - **endogenous level of human capital**
- Solow and **limited technological spillovers**
- Solow with open economy and **factor reallocation** across regions
- Solow with open economy, factor reallocation across countries, and limited technological spillover
- Solow with **two sectors** and factor reallocation across regions (core-periphery, i.e. North-South model)
- Solow with **many intermediate goods**

Endogenous employment rate

Remind standard Solow model:

$$\dot{k} = sf(k, h) - (\delta + g_A + n)k, \quad (1)$$

where

$$k \equiv \frac{K}{AL}, \quad f \equiv F\left(\frac{K}{AL}, h\right) \quad \text{and} \quad f_k > 0, f_{kk} < 0 \quad (2)$$

and s is the exogenous saving/investment rate, h the level of human capital, δ the depreciation rate of physical capital, g_A the growth rate of technological change, but economic theory suggests that:

$$n = \tilde{n}(y) = n(k), \quad \text{with} \quad n' < 0 \quad \text{and} \quad n'' > 0, \quad (3)$$

i.e. the growth rate of employment n negatively depends on the level of income.

Negative relationship between n and y

Why should be a negative relationship between n and y ?

- **quantity-quality trade-off** in the choice of fertility (Becker et al. 1990);
- the **development of financial markets** (children are an asset for your retirement in developing countries) (Cigno 1992);
- the **increasing participation of women** in the fertility decision (Weil 2004).

Solow (1956) discusses how under these assumptions on the relationship between n and y we could observe a **poverty trap** with a cluster of high-fertility versus a cluster of low-fertility regions (countries).

Employment rate and GDP per worker in European regions

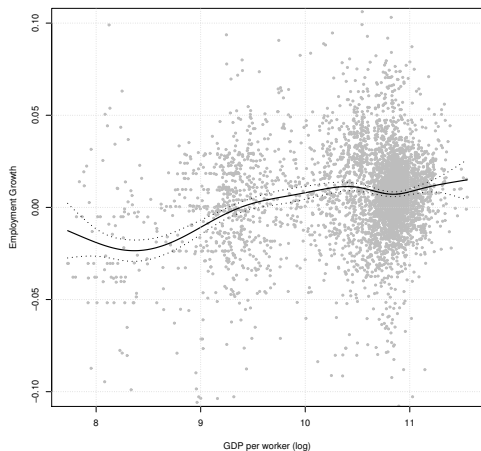


Figura: Growth rate of employment against the level of GDP per worker

Why is there a positive relationship between n and y ?

- **migration** (we will discuss this issue later in the course);
- **efficiency of labour markets** increases with the level of development;

In any case it is crucial to control for the **endogeneity** of growth rate of employment!

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Increasing returns to scale due to structural change

- In general the aggregate output is the result of production in several **sectors**.
- This could implies that even though we have **decreasing/constant returns to scale at sectoral level** at aggregate level we observe **increasing returns to scale**. Simon Kuznets is the most important author in the analysis of this phenomenon.

Consider a two-sector economy, where AGR (agriculture) indicates the first sector, and MAN (manufacturing) indicates the second sector. The aggregate output of region i is given by:

$$Y = Y^{AGR} + p^{MAN} Y^{MAN}, \quad (4)$$

where p^{MAN} is the price of manufacturing goods (price of agricultural goods are normalized to 1).

Aggregation from individual sector

Given two sectoral production functions:

$$Y^{AGR} = F^{AGR} \left(K^{AGR}, A^{AGR} H^{AGR} \right) \text{ and}$$

$$Y^{MAN} = F^{MAN} \left(K^{MAN}, A^{MAN} H^{MAN} \right),$$

where:

- K^{AGR} and K^{MAN} are the stocks of physical capital, H^{AGR} and H^{MAN} are the stocks of human capital, and A^{AGR} A^{MAN} the levels of technological progress in the AGR and MAN sectors respectively;
- F^{AGR} and F^{MAN} satisfy standard properties of production functions: first derivatives positive and second derivatives negative with respect both inputs and constant returns to scale.

Aggregation from individual sector (cont.d)

The aggregate production function is therefore defined as:

$$\begin{aligned}
 Y &= F^{AGR} \left(K^{AGR}, A^{AGR} H^{AGR} \right) + p^{MAN} F^{MAN} \left(K^{MAN}, A^{MAN} H^{MAN} \right) = \\
 &= F(K, AH)
 \end{aligned} \tag{5}$$

where:

$$\begin{aligned}
 K &= K^{AGR} + K^{MAN}, \\
 H &= H^{AGR} + H^{MAN},
 \end{aligned}$$

and A is an index of aggregate technological progress

$\Rightarrow F(K, AH)$ can display **increasing returns to scale**.

This is due to different productivities of inputs in the two sectors: it is the so-called **composition effect**.

Main issues with structural change

- How inputs are allocated to different sectors?
- Which is the plausible shape of aggregate production function?
- Which are the implication for the Solow model?

Allocation of inputs between two sectors

If inputs are paid to their **marginal productivities** as the theory of competitive markets then the allocation of inputs must satisfies:

$$\frac{\partial Y^{AGR}}{\partial H^{AGR}} = w^{AGR} = p^{MAN} \frac{\partial Y^{MAN}}{\partial H^{MAN}} = w^{MAN} = w^E$$

and

$$\frac{\partial Y^{AGR}}{\partial K^{AGR}} = r^{AGR} = p^{MAN} \frac{\partial K^{MAN}}{\partial K^{MAN}} = r^{MAN} = r^E$$

where w is the **real wage** and r the **real interest rate**.

Otherwise there would be an interest of workers to change the sector where they are working and of investors to change the sector where to invest their capital.

Equilibrium in the allocation of human capital

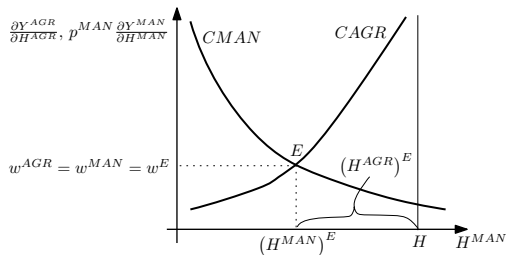


Figura: Allocation of human capital between the two sectors

Some remarks

- Curve C_{MAN} is decreasing in H^{MAN} for the hypothesis of decreasing marginal productivity of human capital decreasing
- Curve C_{AGR} is increasing in H^{MAN} for the hypothesis of decreasing marginal productivity of human capital
($H^{AGR} = H - H^{MAN}$)

Change in the allocation of human capital (cont.d)

Changes in the allocation in the human capital happens for differential changes in

- K^{AGR} and K^{MAN} (sectoral reallocation of physical capital)
- A^{AGR} and A^{MAN} (differential growth rates of technological progress)
- p^{MAN} (preferences of consumers are not homothetic, i.e. there exists a hierarchy across goods)

⇒ all these are the sources of **structural change**

Change in the allocation of human capital

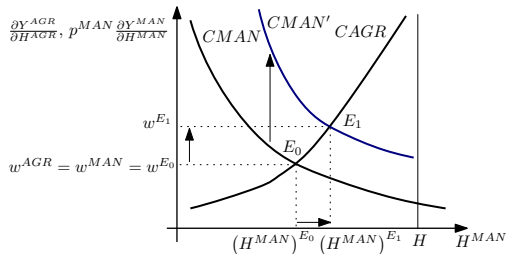


Figura: Increase in the allocation of human capital in sector MAN

An increase in H^{MAN} can be the result of:

- an increase in K^{MAN}
- an increase in A^{MAN}
- an increase in p^{MAN}

The aggregate effect of structural change

- A similar analysis can be made for the allocation of physical capital.
- A rigorous analysis would request to **jointly** consider the allocation of physical and human capital, but the results would be not different of the ones just exposed.
- Under the assumption that sector MAN is more efficient in the utilization of the resources of sector AGR the structural change can produce at aggregate level a function with increasing returns to scale as the result of the **accumulation of inputs**, **technological progress** and **the reallocation** of input from less productive to productive sectors.

The aggregate effect of structural change

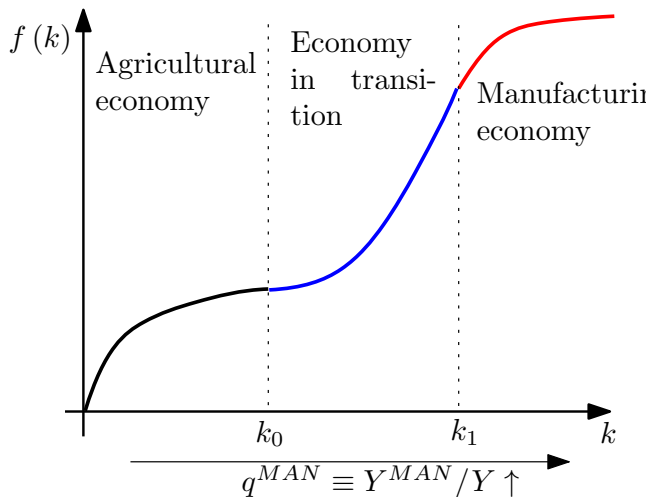


Figura: Aggregate production function with structural change