Quantitative Economics for the Evaluation of the European Policy

Dipartimento di Economia e Management

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Distribution dynamics

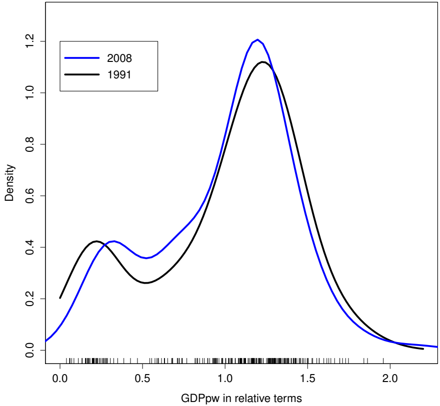


Figura: Estimated distributions of (relative) GDP per worker in 1991 and 2008 in 254 NUTS-2 European regions.

Markov matrix with discrete state space

Define a **set of states** for Y/L: $s = \{s_1, s_2, ..., s_K\}$.
The **probability** of region i to transit from state k to state q with lag τ is defined as:

$$p_{qk} = Pr(Y/L_{i,t+\tau} \in s_q | Y/L_{i,t} \in s_k). \quad (1)$$

If the dynamics of distribution follows a **Markov process**, then the dynamics of the masses of probability related to different states can be represented by:

$$\pi_{t+\tau} = \pi_t P, \quad (2)$$

where $\pi_t = (\pi_{1,t}, ..., \pi_{K,t})$ (row vector) and $\pi_{k,t}$ is the mass of probability of distribution in state k at period t and P, which collect all p_{qk}, is called the **Markov transition matrix** (dimensions KxK).
Under some regularity conditions there exists an **ergodic (equilibrium) distribution** π_∞ such that:

$$\pi_\infty = \pi_\infty P. \quad (3)$$

Distribution dynamics (cont.d)

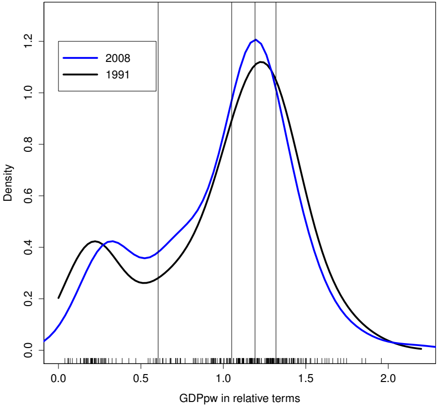


Figura: Estimated distributions of (relative) GDP per worker in 1991 and 2008 in 257 NUTS-2 European regions and state space by the quantiles of distribution

Estimated Markov matrix with discrete state space

State	1	2	3	4	5
Range	0.06-0.60	0.60-1.05	1.05-1.19	1.19-1.32	1.32-2.22

Tabella: Definition of the space states based on the quantile distribution of observations

$t \parallel t+10$	1	2	3	4	5
1	388.00	31.00	0.00	0.00	0.00
2	16.00	331.00	16.00	6.00	0.00
3	0.00	79.00	214.00	83.00	22.00
4	0.00	12.00	140.00	184.00	106.00
5	0.00	0.00	54.00	107.00	266.00

Tabella: Markov matrix for our sample

Estimated Markov transition matrix

$t \parallel t+10$	1	2	3	4	5
1	0.93	0.07	0.00	0.00	0.00
2	0.04	0.90	0.04	0.02	0.00
3	0.00	0.20	0.54	0.21	0.06
4	0.00	0.03	0.32	0.42	0.24
5	0.00	0.00	0.13	0.25	0.62

Tabella: Markov transition matrix for our sample

where the **maximum likelihood estimator** of transition probability is given by:

$$\hat{p}_{qk} = \frac{\text{number of observations starting from state k and arrived to state q}}{\text{total number of observations starting from state k}}$$

Estimated ergodic distribution

From the estimate of $\hat{P}$ we can estimate the ergodic distribution:

$$\hat{\pi}_\infty = \hat{\pi}_\infty \hat{P};$$

in particular:

State	1	2	3	4	5
Mass of probability	0.26	0.45	0.12	0.09	0.07

Tabella: Ergodic distribution

Another definition of states spaces ...

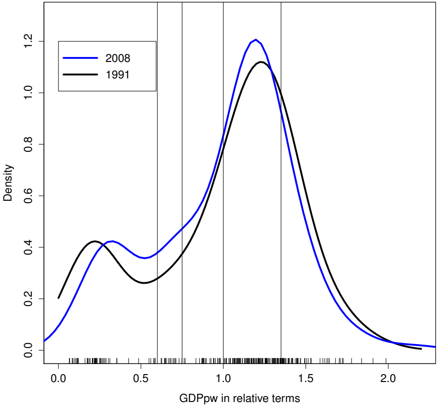


Figura: Estimated distributions of (relative) GDP per worker in 1991 and 2008 in 257 NUTS-2 European regions and alternative state space

Another definition of states spaces ... (cont.d)

State	1	2	3	4	5
Range	0.06-0.60	0.60-0.75	0.75-1.00	1.00-1.35	1.35-2.22

Tabella: Another definition of the space states

$t \parallel t+10$	1	2	3	4	5
1	0.93	0.07	0.00	0.00	0.00
2	0.12	0.67	0.20	0.01	0.00
3	0.00	0.14	0.76	0.10	0.00
4	0.00	0.00	0.09	0.79	0.13
5	0.00	0.00	0.00	0.41	0.59

Tabella: Markov transition matrix for our sample

State	1	2	3	4	5
Mass of probability	0.25	0.16	0.23	0.28	0.09

Tabella: Ergodic distribution

Markov matrix with continuous state space

The definition of the state space may crucially affect the result. Possible solution: **the use of continuous state space**.

Markov matrix with continuous state space becomes a **conditioned distribution**, also denoted **stochastic kernel**:

$$g_\tau \left(Y/L_{i,t+\tau} | Y/L_{i,t} \right) \equiv \frac{f \left(Y/L_{i,t+\tau}, Y/L_{i,t} \right)}{r \left(Y/L_{i,t} \right)}$$

Accordingly the **ergodic distribution** solves:

$$f_\infty (x) = \int_0^\infty g_\tau (x|z) \, f_\infty (z) \, dz, \qquad (4)$$

where x and z are two levels of Y/L, $g_\tau (x|z)$ is the density of x, given z, τ periods ahead, under the constraint

$$\int_0^\infty f_\infty (x) \, dx = 1. \qquad (5)$$

Ergodic distribution with normalized variable

Since in our estimates all variables are normalized with respect to their average, the ergodic distribution must respect the additional constraint:

$$\int_0^\infty f_\infty(x)\, x dx = 1. \tag{6}$$

Then the true ergodic distribution is:

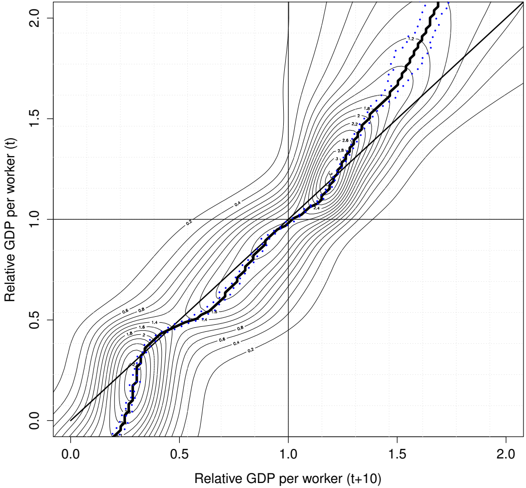
$$f_\infty(x) = \tilde{\mu}_x \tilde{f}_\infty(x), \tag{7}$$

where:

$$\tilde{\mu}_x = \int_0^\infty \tilde{f}_\infty(x)\, x dx \tag{8}$$

and $\tilde{f}_\infty$ satisfies Eqq. (4) and (5).

Stochastic kernel



Median of the stochastic kernel

The bold line represents the **median** of the estimated conditioned distribution $g_\tau\left(Y/L_{i,t+\tau}|Y/L_{i,t}\right)$, denoted by $\text{med}\left(Y/L_{i,t}\right)$.
The median is crucial to understand the dynamics of the **mass of distribution**; in particular:

- if $\text{med}\left(Y/L_{i,t}\right) > Y/L_{i,t}$ then we expect that **locally** the mass is shifting ahead around $Y/L_{i,t}$
- if $\text{med}\left(Y/L_{i,t}\right) < Y/L_{i,t}$ then we expect that **locally** the mass is shifting behind around $Y/L_{i,t}$
- if $\text{med}\left(Y/L_{i,t}\right) = Y/L_{i,t}$ then we expect that **locally** the mass is stable around $Y/L_{i,t}$, i.e. $Y/L_{i,t}$ is a possible **equilibrium**.

Another perspective to see the estimated stochastic kernel is to think about it how a stochastic difference equation:

$$Y/L_{i,t+\tau} = \phi\left(Y/L_{i,t}\right) + \varepsilon_{i,t+\tau} \tag{9}$$

$\Rightarrow$ if around an equilibrium $\text{med}\left(Y/L_{i,t}\right)$ crosses from below, this equilibrium is **stable**, if it crosses from above, this equilibrium is **unstable**.

Estimated ergodic distribution

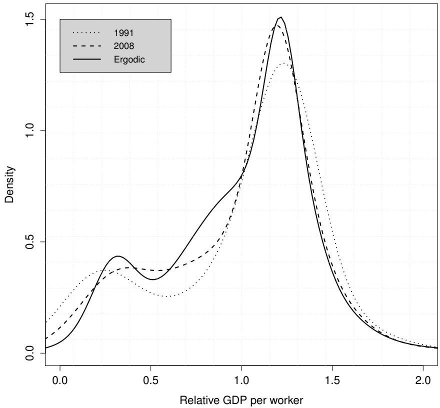


Figura: Estimated ergodic distribution of (relative) GDP per worker in 1991 and 2008 in 257 NUTS-2 European regions