

Quantitative Economics for the Evaluation of the European Policy

Dipartimento di Economia e Management

Co-funded by the
Erasmus+ Programme
of the European Union



Project funded by
European Commission Erasmus + Programme –Jean Monnet Action
Project number 553280-EPP-1-2015-1-IT-EPPJMO-MODULE

Irene Brunetti Davide Fiaschi Angela Parenti¹

October 16, 2017

¹ireneb@ec.unipi.it, davide.fiaschi@unipi.it, and aparenti@ec.unipi.it.

Introduction

- Many interesting features (standard of living, health, etc.) of an economy are related to its level of income, at least in the long run. This is why we mainly focus on GDP per head or GDP per capita \Rightarrow we need a theory of growth of countries/regions
- Many theories are available and generally are related to the supply side of an economy. Other theories on the demand side are available, but generally refer to short and medium run.
- The main differences between these theories are in the assumptions on:
 - the diffusion of technological progress (limited or not);
 - the speed of factor reallocation (mobility);
 - the type of technology (increasing returns to scale); and
 - the type of markets (competitive versus monopolistic markets).

Solow model

Solow model is the prototype of any growth model based on the supply side. Consider the augmented version proposed by Mankiw et al. (1992).

Key ingredients:

- A production function: $Y = F(K, AH)$ assumed with standard properties $F_K > 0$, $F_{KK} < 0$, $F_H > 0$, $F_{HH} < 0$, with constant returns to scale $\lambda Y = F(\lambda K, \lambda AH)$ and $F(0, AH) = 0$;
- A theory on the accumulation of physical capital: $dK/dt \equiv \dot{K} = sY - \delta K$ (s is the exogenous saving/investment rate);
- A theory on the technological change: $\dot{A} = g_A A$ (g_A is exogenous);
- A theory on the accumulation of human capital: hL , where h is the average level of human capital (constant) and L the total number of workers in the the economy; and
- A theory of the growth of workers/population: $\dot{L} = nL$ (n is exogenous)

At the end of day we have:

$$\dot{k} = sf(k, h) - (\delta + g_A + n) k, \quad (1)$$

where

$$k \equiv \frac{K}{AL}, \quad f \equiv F\left(\frac{K}{AL}, h\right) \quad \text{and} \quad f_k > 0, f_{kk} < 0 \quad (2)$$

You can find that k is converging to an equilibrium level given by k^{EQ} positively depending on s and h and negatively on δ , g_A and n .

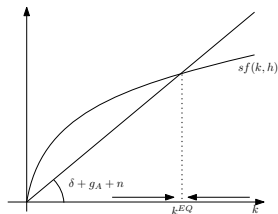


Figura: Equilibrium in Solow model

If you assume that $F(K, AH) = K^\alpha (AH)^{1-\alpha}$ then:

$$\dot{k} = sk^\alpha h^{1-\alpha} - (\delta + g_A + n)k; \quad (3)$$

$$k^{EQ} = h \left(\frac{s}{\delta + g_A + n} \right)^{1/(1-\alpha)}; \quad (4)$$

and

$$y^{EQ} \equiv \frac{Y}{AL} = h \left(\frac{s}{\delta + g_A + n} \right)^{\alpha/(1-\alpha)}. \quad (5)$$

We get a simple theory of the growth of GDP per worker (head) and on the level of income in the long run.

- If y^{EQ} is constant in equilibrium then Y/L is growing at the constant rate of technological progress g_A
- The level of income positively depends on s and h and negatively by δ and n .

We have the additional properties that the growth rate of Y/L , $g_{Y/L}$ is decreasing in the level of y , i.e. more distant you are from the equilibrium higher is your growth rate. Consider:

$$g_k \equiv \frac{\dot{k}}{k} = sk^{(\alpha-1)}h^{1-\alpha} - (\delta + g_A + n); \quad (6)$$

and

$$\frac{\dot{Y}/L}{Y/L} = \alpha g_k + g_A; \quad (7)$$

(remember that $Y/L = Ak^\alpha h^{1-\alpha}$).

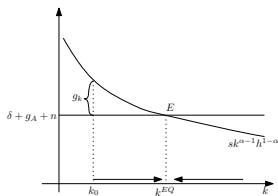


Figura: Relationship between the level of growth and the level of income

We can use Eq. (7) to understand the dynamics of distribution of GDP per worker across European regions.

Two types of convergence:

- Absolute convergence. Key hypothesis: all regions have the same characteristics \Rightarrow convergence at the same level of GDP per worker in the long run and negative relationship between initial level of GDP per worker and growth rate of GDP per worker
- Conditional convergence. Key hypothesis: regions have heterogeneous characteristics \Rightarrow convergence at the same level of GDP per worker in the long run and negative relationship between initial level of GDP per worker and growth rate of GDP per worker but **CONDITIONED** on the differences in the regional characteristics

Absolute convergence

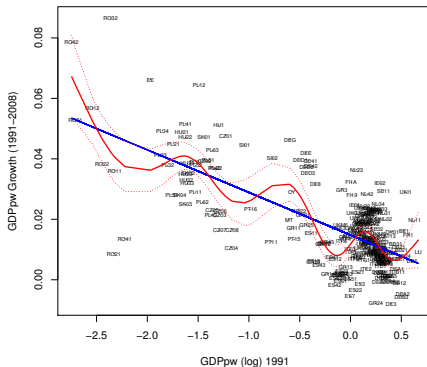


Figura: Absolute convergence in the GDP per worker of 256 European regions. Parametric and nonparametric regression

Econometric model of convergence

Hypothesis of absolute convergence with linear model: $\beta < 0$

$$\overline{g_{Y/L}} = \text{intercept} + \beta \log(Y/L_{i,1991}) + \epsilon_i \quad (8)$$

	Estimate	Std. Error	t-Stat.	P-value
(Intercept)	0.0148	0.0007	22.01	0.0000
β	-0.0141	0.0009	-16.17	0.0000
Res.se=0.01044 (255) DF				
R-squared=0.5063, Adj.R-squared=0.5044				
F-stat.=261.5 (1,255) DF, p-value=< $2e^{-16}$				

Econometric model of convergence (cont.d)

Hypothesis of absolute convergence with a nonparametric model: $\phi' < 0$

$$\overline{g_{Y/L}} = \text{intercept} + \phi(\log(Y/L_{i,1991})) + \epsilon_i \quad (9)$$

Parametric coeff.:	Estimate	Std. Error	t-Stat	P-value
(Intercept)	0.0175179	0.0006103	28.7	$< 2e^{-16}$ ***
Smooth terms:	edf	Ref.df	F	p-value
$\phi(\cdot)$	8.722	8.978	37.97	$< 2e^{-16}$ ***
R-sq.(adj)=0.565; Dev.expl.=58%				
GCV= $9.948e^{-05}$; Scale est.= $9.5717e^{-05}$; n=257				

Conditional convergence

Hypothesis of conditional convergence with linear model: $\beta_0 < 0$

$$\overline{g_{Y/L}} = \text{intercept} + \beta_0 \log(Y/L_{i,1991}) + \beta_1 \bar{s} + \beta_2 \bar{n} + \beta_3 \bar{h} + \epsilon_i \quad (10)$$

	Estimate	Std. Error	t-Stat.	P-value
(Intercept)	-0.0929	0.0123	-7.53	0.0000
β_0	-0.0154	0.0011	-14.57	0.0000
β_1	0.0027	0.0029	0.93	0.3532
β_2	-0.0146	0.0034	-4.31	0.0000
β_3	0.0204	0.0024	8.57	0.0000
Res.se=0.008956 (255) DF				
R-squared=0.6411, Adj.R-squared=0.6354				
F-stat.=112.6 (1,255) DF, p-value=< 2e ⁻¹⁶				

Conditional convergence

Hypothesis of conditional convergence with a nonparametric model:

$$\phi'_0 < 0$$

$$\overline{g_{Y/L}} = \text{intercept} + \phi_0 (\log(Y/L_{i,1991})) + \phi_1(\bar{s}) + \phi_2(\bar{n}) + \phi_3(\bar{h}) + \epsilon_i \quad (11)$$

Parametric coeff.:	Estimate	Std. Error	t-Stat	P-value
(Intercept)	0.0175179	0.0004611	37.99	$< 2e^{-16}$ ***
Smooth terms:	edf	Ref.df	F	p-value
$\phi_0(\cdot)$	8.641	8.963	39.175	$< 2e^{-16}$ ***
$\phi_1(\cdot)$	5.392	6.582	1.722	0.109
$\phi_2(\cdot)$	8.595	8.95	5.644	$< 2e^{-16}$ ***
$\phi_3(\cdot)$	1.235	1.434	80	$< 2e^{-16}$ ***
R-sq.(adj)=0.752; Dev.expl.=77.5%				
GCV=6.0497e ⁻⁰⁵ ; Scale est.=5.4645e ⁻⁰⁵ ; n=257				

Distribution Dynamics

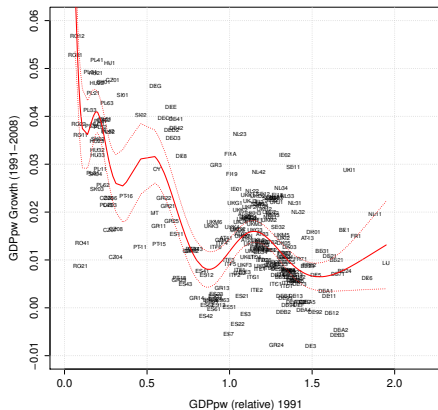


Figura: Absolute convergence in the GDP per worker of 256 European regions. Parametric and nonparametric regression.

Distribution Dynamics (cont.d)

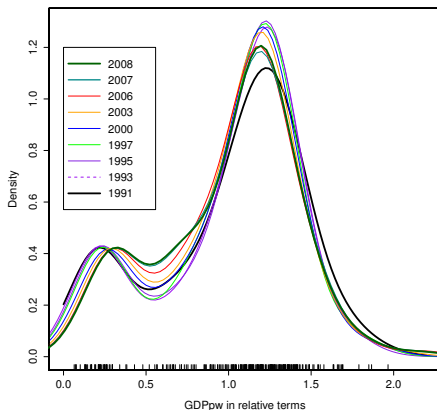


Figura: Estimated distributions of (relative) GDP per worker from 1991 to 2008 in 254 NUTS-2 European regions.