

Quantitative Economics for the Evaluation of the European Policy

Dipartimento di Economia e Management

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Econometric model of convergence

Hypothesis of absolute convergence with linear model: $\beta < 0$

$$\overline{g_{Y/L}} = \text{intercept} + \beta \log(Y/L_{i,1991}) + \epsilon_i \quad (1)$$

	Estimate	Std. Error	t-Stat.	P-value
(Intercept)	0.0148	0.0007	22.01	0.0000
β	-0.0141	0.0009	-16.17	0.0000
Res.se=0.01044 (255) DF R-squared=0.5063, Adj.R-squared=0.5044 F-stat.=261.5 (1,255) DF, p-value=< $2e^{-16}$				

Econometric model of convergence (cont.d)

Hypothesis of absolute convergence with a nonparametric model: $\phi' < 0$

$$\overline{g_{Y/L}} = \text{intercept} + \phi(\log(Y/L_{i,1991})) + \epsilon_i \quad (2)$$

Parametric coeff.:	Estimate	Std. Error	t-Stat	P-value
(Intercept)	0.0175179	0.0006103	28.7	$< 2e^{-16}$ ***
Smooth terms:	edf	Ref.df	F	p-value
$\phi(\cdot)$	8.722	8.978	37.97	$< 2e^{-16}$ ***
R-sq.(adj)=0.565; Dev.expl.=58%				
GCV= $9.948e^{-05}$; Scale est.= $9.5717e^{-05}$; n=257				

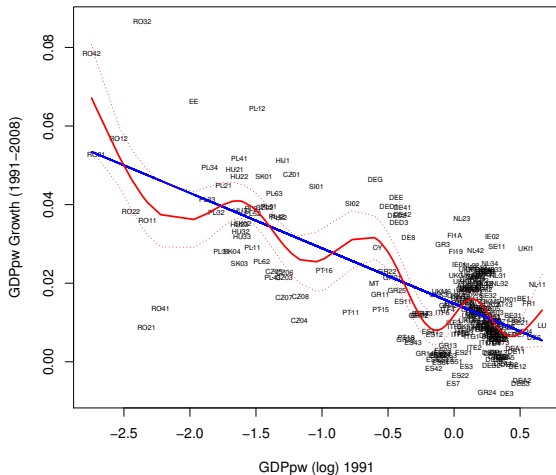


Figura: Absolute convergence in the GDP per worker. Parametric and nonparametric regression

Conditional convergence

Hypothesis of conditional convergence with linear model: $\beta_0 < 0$

$$\overline{g_{Y/L}} = \text{intercept} + \beta_0 \log(Y/L_{i,1991}) + \beta_1 \bar{s} + \beta_2 \bar{n} + \beta_3 \bar{h} + \epsilon_i \quad (3)$$

	Estimate	Std. Error	t-Stat.	P-value
(Intercept)	-0.0929	0.0123	-7.53	0.0000
β_0	-0.0154	0.0011	-14.57	0.0000
β_1	0.0027	0.0029	0.93	0.3532
β_2	-0.0146	0.0034	-4.31	0.0000
β_3	0.0204	0.0024	8.57	0.0000
Res.s.e. = 0.008956 (255) DF R-squared=0.6411, Adj.R-squared=0.6354 F-stat.=112.6 (1,255) DF, p-value=< $2e^{-16}$				

Conditional convergence

Hypothesis of conditional convergence with a nonparametric model:

$$\phi'_0 < 0$$

$$\overline{g_{Y/L}} = \text{intercept} + \phi_0 (\log(Y/L_{i,1991})) + \phi_1 (\bar{s}) + \phi_2 (\bar{n}) + \phi_3 (\bar{h}) + \epsilon_i \quad (4)$$

Parametric coeff.:	Estimate	Std. Error	t-Stat	P-value
(Intercept)	0.0175179	0.0004611	37.99	$< 2e^{-16}$ ***
Smooth terms:	edf	Ref.df	F	p-value
$\phi_0(\cdot)$	8.641	8.963	39.175	$< 2e^{-16}$ ***
$\phi_1(\cdot)$	5.392	6.582	1.722	0.109
$\phi_2(\cdot)$	8.595	8.95	5.644	$< 2e^{-16}$ ***
$\phi_3(\cdot)$	1.235	1.434	80	$< 2e^{-16}$ ***
R-sq.(adj)=0.752; Dev.expl.=77.5%				
GCV=6.0497e ⁻⁰⁵ ; Scale est.=5.4645e ⁻⁰⁵ ; n=257				

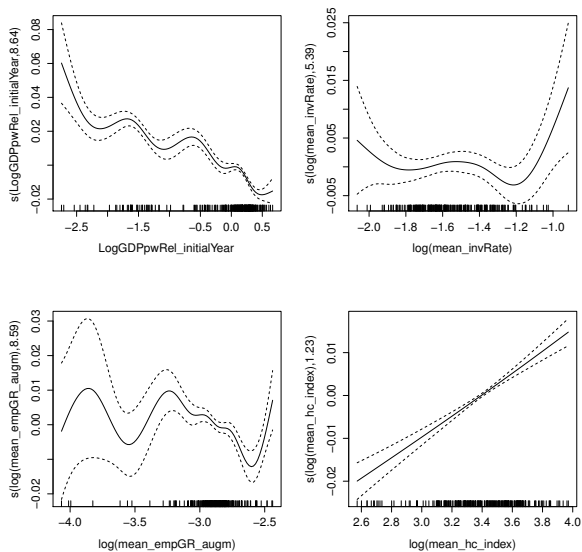


Figure: Estimate of generalized additive model.

Another type of convergence ... σ -convergence

Variance of the log of the income per worker

$$\sigma_t^2 = \frac{\sum_i^N [\log(Y/L_{i,t}) - \mu_t]^2}{N} \quad (5)$$

Mean of the log of the income per worker

$$\mu_t = \frac{\sum_i^N \log(Y/L_{i,t})}{N} \quad (6)$$

Then:

$$\sigma_t = \text{intercept} + \gamma t + \eta_t \quad (7)$$

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	19.2588	0.7099	27.13	0.0000
γ	-0.0093	0.0004	-26.18	0.0000

Residual s.e.: 0.007814 on 16 d.f.

R^2 : 0.98, Adj. R^2 : 0.98

F-stat.: 685.2 on 1 and 16 DF, p-value: 1.458e-14

σ -convergence (con.d)

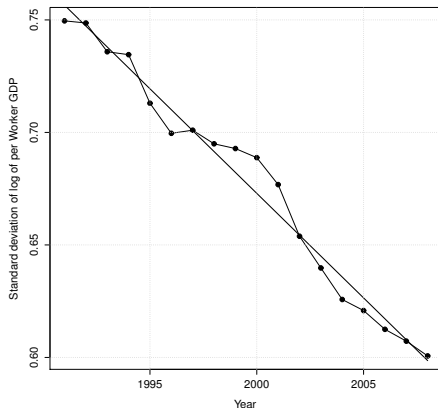


Figura: σ -convergence in the log of GDP per worker of 256 European regions.

σ -convergence (con.d)

But suppose that there income follows (with $b \in (0, 1)$ to ensure convergence)

$$\log(Y/L_{i,t}) = a + (1 - b) \log(Y/L_{i,t-1}) + u_{i,t} \quad (8)$$

from which:

$$\sigma_t^2 = (1 - b)^2 \sigma_{t-1}^2 + \sigma_u^2 \quad (9)$$

and therefore for $t \rightarrow \infty$:

$$\sigma_\infty^2 = \frac{\sigma_u^2}{1 - (1 - b)^2} \quad (10)$$

\Rightarrow variance can decrease or increase according to the relationship between σ_t^2 and σ_∞^2 even though there is absolute convergence.

Galton fallacy: absolute convergence does not imply decreasing variance of distribution

Distribution Dynamics

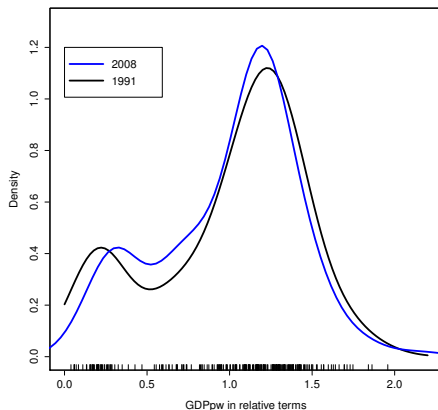


Figura: Estimated distributions of (relative) GDP per worker in 1991 and 2008 in 254 NUTS-2 European regions.

Markov matrix with discrete state space

Define a **set of states** for Y/L : $s = \{s_1, s_2, \dots, s_K\}$.

The **probability** of region i to transit from state k to state q with lag τ is defined as:

$$p_{qk} = Pr(Y/L_{i,t} \in s_q | Y/L_{i,t-\tau} \in s_k). \quad (11)$$

If the dynamics of distribution follows a **Markov process**, then the dynamics of the masses of probability related to different states can be represented by:

$$\pi_t = \pi_{t-\tau} P, \quad (12)$$

where $\pi_t = (\pi_{1,t}, \dots, \pi_{K,t})$ (row vector) and $\pi_{k,t}$ is the mass of probability of distribution in state k at period t and P , which collect all p_{qk} , is called the **Markov transition matrix** (dimensions $K \times K$).

Under some regularity conditions there exists an **ergodic (equilibrium) distribution** π_∞ such that:

$$\pi_\infty = \pi_\infty P. \quad (13)$$

Distribution Dynamics (cont.d)

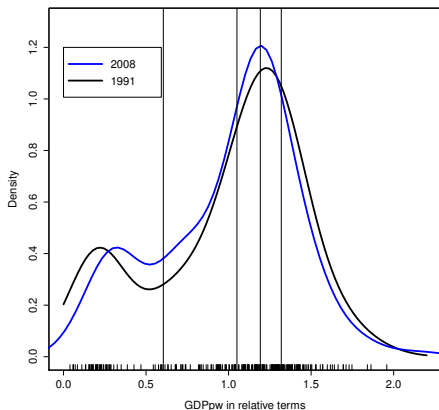


Figura: Estimated distributions of (relative) GDP per worker in 1991 and 2008 in 254 NUTS-2 European regions and state space by the quantiles of distribution

Markov matrix with discrete state space

State	1	2	3	4	5
Range	0.06-0.60	0.60-1.05	1.05-1.19	1.19-1.32	1.32-2.22

Tabella: Definition of the space states based on the quantile distribution of actual observations

$t - 1 \parallel t$	1	2	3	4	5
1	388.00	31.00	0.00	0.00	0.00
2	16.00	331.00	16.00	6.00	0.00
3	0.00	79.00	214.00	83.00	22.00
4	0.00	12.00	140.00	184.00	106.00
5	0.00	0.00	54.00	107.00	266.00

Tabella: Markov matrix for our sample

Markov transition matrix

$t - 1 \parallel t$	1	2	3	4	5
1	0.93	0.07	0.00	0.00	0.00
2	0.04	0.90	0.04	0.02	0.00
3	0.00	0.20	0.54	0.21	0.06
4	0.00	0.03	0.32	0.42	0.24
5	0.00	0.00	0.13	0.25	0.62

Tabella: Markov transition matrix for our sample

State	1	2	3	4	5
Mass of probability	0.26	0.45	0.12	0.09	0.07

Tabella: Ergodic distribution