

# Quantitative Economics for the Evaluation of the European Policy

Dipartimento di Economia e Management

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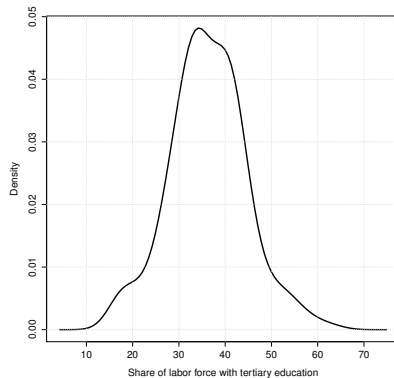
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- Solow model with poverty trap or better **multiple equilibria** (but why only two?)
  - ☑ **endogenous investment rate**
  - ☑ **endogenous growth rate of population/employment**
  - ☑ **increasing returns to scale (change in output composition)**
  - ⇨ **endogenous level of human capital**
- Solow and **limited technological spillovers**
- Solow with open economy and **factor reallocation** across regions
- Solow with open economy, factor reallocation across countries, and limited technological spillover
- Solow with **two sectors** and factor reallocation across regions (core-periphery, i.e. North-South model)
- Solow with **many intermediate goods**

# Human capital in European regions

Could human capital explain the differences in GDP per worker in European regions?



**Figura:** Distribution of the share of employment with tertiary education in European regions

# Main issues about human capital

Main issues:

- How human capital is accumulated
- How is possible to measure it
- How is possible to favour the accumulation of human capital?

# The theory of human capital

Remind standard Solow model:

$$\dot{k} = sf(k, h) - (\delta + g_A + n) k, \quad (1)$$

where

$$k \equiv \frac{K}{AL}, \quad f \equiv F\left(\frac{K}{AL}, h\right) \equiv f(k, h) \quad \text{and} \quad f_k > 0, f_{kk} < 0 \quad (2)$$

and  $s$  and  $n$  are the exogenous saving/investment rate and growth rate of employment,  $h$  the level of human capital,  $\delta$  the depreciation rate of physical capital, and  $g_A$  the growth rate of technological change.

Differences in the level of GDP per worker due to differences in human capital

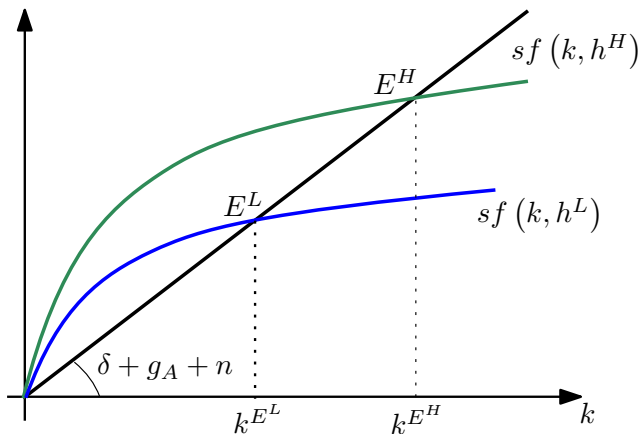


Figura:

⇒ Now we want to formulate a theory of the level (dynamics) of  $h$

Suppose that the accumulation of human capital can be expressed as:

$$\dot{h} = \Phi(h, y, s_h y, CN) - \delta_h, \quad (3)$$

with  $\Phi_h > 0$ ,  $\Phi_y > 0$ , and  $\Phi_{s_h} > 0$ .

Why these explanatory variable?

- $h$ : **spillover effects** deriving from living in a “skilled” environment (Lucas, Durlauf, Brock and Durlauf, etc.)
- $y$ : **learning by doing** (Arrow and Lucas)
- $s_h$ : **financial investment in education/human capital** (Lucas, Galor and Zeira)
- $CN$  other determinants related to **cultural norms** (gender discrimination, etc.) (Weil)
- $\delta_h$ : depreciation of human capital due to various factors, among which the most important is the technological progress

**Assumption:**  $\Phi(\cdot)$  is homogeneous of degree one in the first three arguments

Example:

$$\Phi(h, y, s_h y, CN) = h^\beta y^\gamma s_h^{1-\beta-\gamma} \quad (4)$$

Then:

$$\dot{h} = h\Phi\left(1, \frac{y}{h}, \frac{s_h y}{h}, CN\right) - \delta_h h = h\Phi\left(1, \frac{f(k, h)}{h}, \frac{s_h f(k, h)}{h}, CN\right) - h\delta_h \quad (5)$$

from which:

$$\frac{\dot{h}}{h} = \Phi\left(1, \frac{f(k, h)}{h}, \frac{s_h f(k, h)}{h}, CN\right) - \delta_h \quad (6)$$

$\Rightarrow$  the dynamics of  $\dot{h}/h$  crucially depends on the **average product of human capital**  $f(k, h)/h$ .



Two possibilities:

- If we consider technology where average product of human capital is bounded from below, i.e. it cannot go under a certain threshold then the accumulation of human capital **ALONE** can generate long-run growth (Lucas, Glaser, etc.) and differences in human capital generates differences in growth rates.
- If we take the usual Cobb-Douglas production function  $y = k^\alpha h^{1-\alpha}$  then  $f(k, h)/h$  is decreasing in  $h$  and converging to zero. Then we have to consider the **joint dynamics** of  $k$  and  $h$  to understand the overall dynamics and the level of equilibrium of income.

To study the joint dynamics of  $k$  and  $h$  consider the special case of Codd-Douglas production function. Then:

$$\frac{\dot{k}}{k} = s \frac{f(k)}{k} - (\delta + n + g_A) = s \left( \frac{k}{h} \right)^{\alpha-1} - (\delta + n + g_A) \quad (7)$$

and

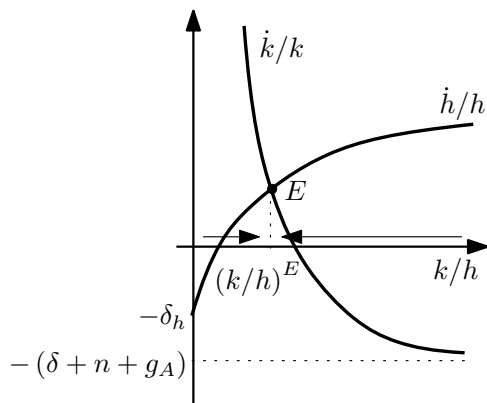
$$\frac{\dot{h}}{h} = \Phi \left( 1, \left( \frac{k}{h} \right)^{1-\alpha}, s_h \left( \frac{k}{h} \right)^{1-\alpha}, CN \right) - \delta_h \quad (8)$$

$\Rightarrow$  the crucial variable for the dynamics is the dynamics of the ratio  $k/h$ .

$$\frac{\dot{k/h}}{k/h} = \frac{\dot{k}}{k} - \frac{\dot{h}}{h} = \quad (9)$$

$$= s \left( \frac{k}{h} \right)^{\alpha-1} - (\delta + n + g_A) + \quad (10)$$

$$- \left[ \Phi \left( 1, \left( \frac{k}{h} \right)^{1-\alpha}, s_h \left( \frac{k}{h} \right)^{1-\alpha}, CN \right) - \delta_h \right] \quad (11)$$



An increase in  $(k/h)^E$  can be the result of:

- an increase in  $s$
- a decrease in  $n$
- a decrease in  $s_h$
- an increase in  $\delta_h$
- an increase in  $\alpha$

Figura: Dynamics of model with physical and human capital