

# Quantitative Economics for the Evaluation of the European Policy

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- Solow model with poverty trap or better **multiple equilibria** (but why only two?)
  - ☑ **endogenous investment rate**
    - ⇨ **endogenous growth rate of population/employment**
      - **increasing returns to scale (change in output composition)**
      - **endogenous level of human capital**
- Solow and **limited technological spillovers**
- Solow with open economy and **factor reallocation** across regions
- Solow with open economy, factor reallocation across countries, and limited technological spillover
- Solow with **two sectors** and factor reallocation across regions (core-periphery, i.e. North-South model)
- Solow with **many intermediate goods**

# Endogenous employment rate

Remind standard Solow model:

$$\dot{k} = sf(k, h) - (\delta + g_A + n)k, \quad (1)$$

where

$$k \equiv \frac{K}{AL}, \quad f \equiv F\left(\frac{K}{AL}, h\right) \quad \text{and} \quad f_k > 0, f_{kk} < 0 \quad (2)$$

and  $s$  is the exogenous saving/investment rate,  $h$  the level of human capital,  $\delta$  the depreciation rate of physical capital,  $g_A$  the growth rate of technological change, but economic theory suggests that:

$$n = \tilde{n}(y) = n(k), \quad \text{with} \quad n' < 0 \quad \text{and} \quad n'' > 0, \quad (3)$$

i.e. the growth rate of employment  $n$  negatively depends on the level of income.

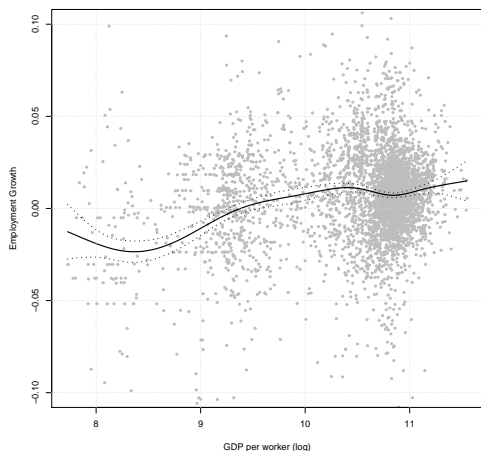
# Negative relationship between $n$ and $y$

Why should be a negative relationship between  $n$  and  $y$ ?

- **quantity-quality trade-off** in the choice of fertility (Becker et al. 1990);
- the **development of financial markets** (children are an asset for your retirement in developing countries) (Cigno 1992);
- the **increasing participation of women** in the fertility decision (Weil 2004).

Solow (1956) discusses how under these assumptions on the relationship between  $n$  and  $y$  we could observe a **poverty trap** with a cluster of high-fertility versus a cluster of low-fertility regions (countries).

# Employment rate and GDP per worker in European regions



**Figura:** Growth rate of employment against the level of GDP per worker

Why is there a positive relationship between  $n$  and  $y$ ?

- **migration** (we will discuss this issue later in the course);
- **efficiency of labour markets** increases with the level of development;

In any case it is crucial to control for the **endogeneity** of growth rate of employment!

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## Increasing returns to scale due to structural change

- In general the aggregate output is the result of production in several **sectors**.
- This could implies that even though we have **decreasing/constant returns to scale at sectoral level** at aggregate level we observe **increasing returns to scale**. Simon Kuznets is the most important author in the analysis of this phenomenon.

Consider a two-sector economy, where AGR (agriculture) indicates the first sector, and MAN (manufacturing) indicates the second sector. The aggregate output of region  $i$  is given by:

$$Y = Y^{AGR} + p^{MAN} Y^{MAN}, \quad (4)$$

where  $p^{MAN}$  is the price of manufacturing goods (price of agricultural goods are normalized to 1).

# Aggregation from individual sector

Given two sectoral production functions:

$$Y^{AGR} = F^{AGR} \left( K^{AGR}, A^{AGR} H^{AGR} \right) \text{ and}$$

$$Y^{MAN} = F^{MAN} \left( K^{MAN}, A^{MAN} H^{MAN} \right),$$

where:

- $K^{AGR}$  and  $K^{MAN}$  are the stocks of physical capital,  $H^{AGR}$  and  $H^{MAN}$  are the stocks of human capital, and  $A^{AGR}$   $A^{MAN}$  the levels of technological progress in the AGR and MAN sectors respectively;
- $F^{AGR}$  and  $F^{MAN}$  satisfy standard properties of production functions: first derivatives positive and second derivatives negative with respect both inputs and constant returns to scale.



## Aggregation from individual sector (cont.d)

The aggregate production function is therefore defined as:

$$\begin{aligned} Y &= F^{AGR} \left( K^{AGR}, A^{AGR} H^{AGR} \right) + p^{MAN} F^{MAN} \left( K^{MAN}, A^{MAN} H^{MAN} \right) = \\ &= F(K, AH) \end{aligned} \quad (5)$$

where:

$$\begin{aligned} K &= K^{AGR} + K^{MAN}, \\ H &= H^{AGR} + H^{MAN}, \end{aligned}$$

and  $A$  is an index of aggregate technological progress

$\Rightarrow F(K, AH)$  can display **increasing returns to scale**.

This is due to different productivities of inputs in the two sectors: it is the so-called **composition effect**.

# Main issues with structural change

- How inputs are allocated to different sectors?
- Which is the plausible shape of aggregate production function?
- Which are the implication for the Solow model?

# Allocation of inputs between two sectors

If inputs are paid to their **marginal productivities** as the theory of competitive markets then the allocation of inputs must satisfies:

$$\frac{\partial Y^{AGR}}{\partial H^{AGR}} = w^{AGR} = p^{MAN} \frac{\partial Y^{MAN}}{\partial H^{MAN}} = w^{MAN} = w^E$$

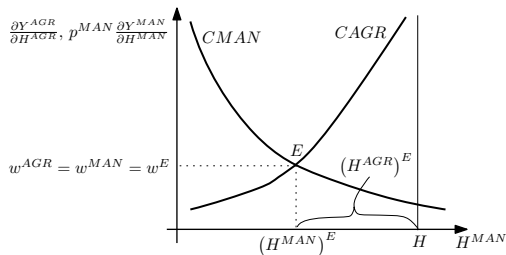
and

$$\frac{\partial Y^{AGR}}{\partial K^{AGR}} = r^{AGR} = p^{MAN} \frac{\partial K^{MAN}}{\partial K^{MAN}} = r^{MAN} = r^E$$

where  $w$  is the **real wage** and  $r$  the **real interest rate**.

Otherwise there would be an interest of workers to change the sector where they are working and of investors to change the sector where to invest their capital.

# Equilibrium in the allocation of human capital



**Figura:** Allocation of human capital between the two sectors

## Some remarks

- Curve  $CMAN$  is decreasing in  $H^{MAN}$  for the hypothesis of decreasing marginal productivity of human capital decreasing
- Curve  $CAGR$  is increasing in  $H^{MAN}$  for the hypothesis of decreasing marginal productivity of human capital  
 $(H^{AGR} = H - H^{MAN})$

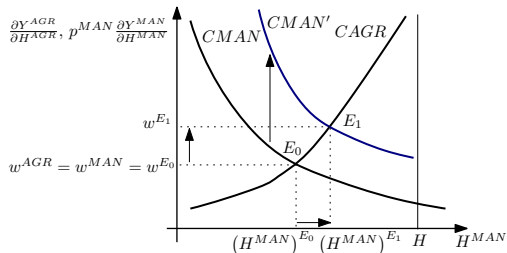
# Change in the allocation of human capital (cont.d)

Changes in the allocation in the human capital happens for differential changes in

- $K^{AGR}$  and  $K^{MAN}$  (sectoral reallocation of physical capital)
- $A^{AGR}$  and  $A^{MAN}$  (differential growth rates of technological progress)
- $p^{MAN}$  (preferences of consumers are not homothetic, i.e. there exists a hierarchy across goods)

⇒ all these are the sources of **structural change**

## Change in the allocation of human capital



**Figura:** Increase in the allocation of human capital in sector MAN

An increase in  $H^{MAN}$  can be the result of:

- an increase in  $K^{MAN}$
- an increase in  $A^{MAN}$
- an increase in  $p^{MAN}$

# The aggregate effect of structural change

- A similar analysis can be made for the allocation of physical capital.
- A rigorous analysis would request to **jointly** consider the allocation of physical and human capital, but the results would be not different of the ones just exposed.
- Under the assumption that sector MAN is more efficient in the utilization of the resources of sector AGR the structural change can produce at aggregate level a function with increasing returns to scale as the result of the **accumulation of inputs**, **technological progress** and **the reallocation** of input from less productive to productive sectors.

## The aggregate effect of structural change

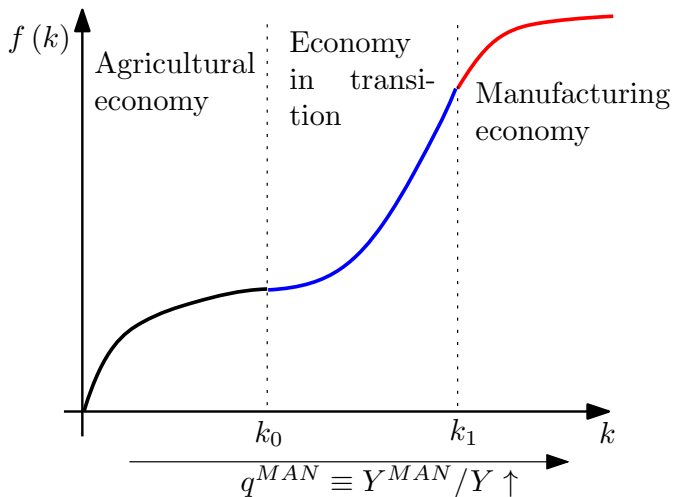


Figura: Aggregate production function with structural change