

# Quantitative Economics for the Evaluation of the European Policy

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# Introduction

- Many interesting features (standard of living, health, etc.) of an economy are related to its level of income, at least in the long run. This is why we mainly focus on GDP per head or GDP per capita  $\Rightarrow$  we need a theory of growth of countries/regions
- Many theories are available and generally are related to the supply side of an economy. Other theories on the demand side are available, but generally refer to short and medium run.
- The main differences between these theories are in the assumptions on:
  - the diffusion of technological progress (limited or not);
  - the speed of factor reallocation (mobility);
  - the type of technology (increasing returns to scale); and
  - the type of markets (competitive versus monopolistic markets).

# Solow model

Solow model is the prototype of any growth model based on the supply side. Consider the augmented version proposed by Mankiw et al. (1992).

Key ingredients:

- A production function:  $Y = F(K, AH)$  assumed with standard properties  $F_K > 0$ ,  $F_{KK} < 0$ ,  $F_H > 0$ ,  $F_{HH} < 0$ , with constant returns to scale  $\lambda Y = F(\lambda K, \lambda AH)$  and  $F(0, AH) = 0$ ;
- A theory on the accumulation of physical capital:  
 $dK/dt \equiv \dot{K} = sY - \delta K$  ( $s$  is the exogenous saving/investment rate);
- A theory on the technological change:  $\dot{A} = g_A A$  ( $g_A$  is exogenous);
- A theory on the accumulation of human capital:  $hL$ , where  $h$  is the average level of human capital (constant) and  $L$  the total number of workers in the the economy; and
- A theory of the growth of workers/population:  $\dot{L} = nL$  ( $n$  is exogenous)

At the end of day we have:

$$\dot{k} = sf(k, h) - (\delta + g_A + n) k, \quad (1)$$

where

$$k \equiv \frac{K}{AL}, \quad f \equiv F\left(\frac{K}{AL}, h\right) \quad \text{and} \quad f_k > 0, f_{kk} < 0 \quad (2)$$

You can find that  $k$  is converging to an equilibrium level given by  $k^{EQ}$  positively depending on  $s$  and  $h$  and negatively on  $\delta$ ,  $g_A$  and  $n$ .

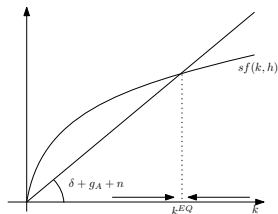


Figura: Equilibrium in Solow model

If you assume that  $F(K, AH) = K^\alpha (AH)^{1-\alpha}$  then:

$$\dot{k} = sk^\alpha h^{1-\alpha} - (\delta + g_A + n)k; \quad (3)$$

$$k^{EQ} = h \left( \frac{s}{\delta + g_A + n} \right)^{1/(1-\alpha)}; \quad (4)$$

and

$$y^{EQ} \equiv \frac{Y}{AL} = h \left( \frac{s}{\delta + g_A + n} \right)^{\alpha/(1-\alpha)}. \quad (5)$$

We get a simple theory of the growth of GDP per worker (head) and on the level of income in the long run.

- If  $y^{EQ}$  is constant in equilibrium then  $Y/L$  is growing at the constant rate of technological progress  $g_A$
- The level of income positively depends on  $s$  and  $h$  and negatively by  $\delta$  and  $n$ .

We have the additional properties that the growth rate of  $Y/L$ ,  $g_{Y/L}$  is decreasing in the level of  $y$ , i.e. more distant you are from the equilibrium higher is your growth rate. Consider:

$$g_k \equiv \frac{\dot{k}}{k} = sk^{(\alpha-1)}h^{1-\alpha} - (\delta + g_A + n); \quad (6)$$

and

$$\frac{\dot{Y}/L}{Y/L} = \alpha g_k + g_A; \quad (7)$$

(remember that  $Y/L = Ak^\alpha h^{1-\alpha}$ ).

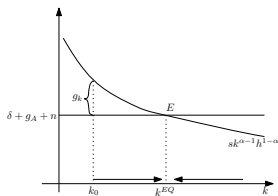


Figura: Relationship between the level of growth and the level of income

We can use Eq. (7) to understand the dynamics of distribution of GDP per worker across European regions.

Two types of convergence:

- Absolute convergence. Key hypothesis: all regions have the same characteristics  $\Rightarrow$  convergence at the same level of GDP per worker in the long run and negative relationship between initial level of GDP per worker and growth rate of GDP per worker
- Conditional convergence. Key hypothesis: regions have heterogeneous characteristics  $\Rightarrow$  convergence at the same level of GDP per worker in the long run and negative relationship between initial level of GDP per worker and growth rate of GDP per worker but **CONDITIONED** on the differences in the regional characteristics

# Absolute convergence

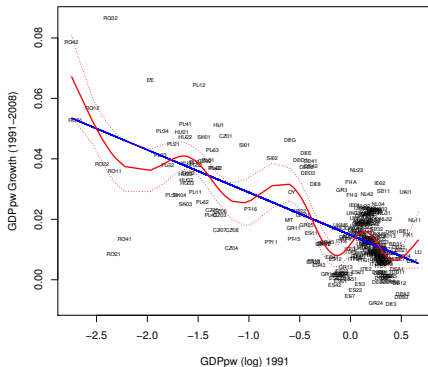


Figura: Absolute convergence in the GDP per worker of 256 European regions. Parametric and nonparametric regression



# Econometric model of convergence

Hypothesis of absolute convergence with linear model:  $\beta < 0$

$$\overline{g_{Y/L}} = \text{intercept} + \beta \log(Y/L_{i,1991}) + \epsilon_i \quad (8)$$

|  | Estimate | Std. Error | t-Stat. | P-value |
|--|----------|------------|---------|---------|
| (Intercept)                                    | 0.0148   | 0.0007     | 22.01   | 0.0000  |
| $\beta$  | -0.0141  | 0.0009     | -16.17  | 0.0000  |
| Res.se=0.01044 (255) DF                        |          |            |         |         |
| R-squared=0.5063, Adj.R-squared=0.5044         |          |            |         |         |
| F-stat.=261.5 (1,255) DF, p-value=< $2e^{-16}$ |          |            |         |         |

## Econometric model of convergence (cont.d)

Hypothesis of absolute convergence with a nonparametric model:  $\phi' < 0$

$$\overline{g_{Y/L}} = \text{intercept} + \phi(\log(Y/L_{i,1991})) + \epsilon_i \quad (9)$$

| Parametric coeff.:  | Estimate  | Std. Error | t-Stat | P-value          |
|---|-----------|------------|--------|------------------|
| (Intercept)   | 0.0175179 | 0.0006103  | 28.7   | $< 2e^{-16}$ *** |
| Smooth terms:   | edf       | Ref.df     | F      | p-value          |
| $\phi(\cdot)$   | 8.722     | 8.978      | 37.97  | $< 2e^{-16}$ *** |
| R-sq.(adj)=0.565; Dev.expl.=58%                           |           |            |        |                  |
| GCV= $9.948e^{-05}$ ; Scale est.= $9.5717e^{-05}$ ; n=257 |           |            |        |                  |

# Conditional convergence

Hypothesis of conditional convergence with linear model:  $\beta_0 < 0$

$$\overline{g_{Y/L}} = \text{intercept} + \beta_0 \log(Y/L_{i,1991}) + \beta_1 \bar{s} + \beta_2 \bar{n} + \beta_3 \bar{h} + \epsilon_i \quad (10)$$

|   | Estimate | Std. Error | t-Stat. | P-value |
|---|----------|------------|---------|---------|
| (Intercept)   | -0.0929  | 0.0123     | -7.53   | 0.0000  |
| $\beta_0$   | -0.0154  | 0.0011     | -14.57  | 0.0000  |
| $\beta_1$   | 0.0027   | 0.0029     | 0.93    | 0.3532  |
| $\beta_2$   | -0.0146  | 0.0034     | -4.31   | 0.0000  |
| $\beta_3$   | 0.0204   | 0.0024     | 8.57    | 0.0000  |
| Res.se=0.008956 (255) DF                              |          |            |         |         |
| R-squared=0.6411, Adj.R-squared=0.6354                |          |            |         |         |
| F-stat.=112.6 (1,255) DF, p-value=< 2e <sup>-16</sup> |          |            |         |         |

# Conditional convergence

Hypothesis of conditional convergence with a nonparametric model:

$$\phi'_0 < 0$$

$$\overline{g_{Y/L}} = \text{intercept} + \phi_0 (\log(Y/L_{i,1991})) + \phi_1(\bar{s}) + \phi_2(\bar{n}) + \phi_3(\bar{h}) + \epsilon_i \quad (11)$$

| Parametric coeff.:   | Estimate  | Std. Error | t-Stat | P-value          |
|--|-----------|------------|--------|------------------|
| (Intercept)  | 0.0175179 | 0.0004611  | 37.99  | $< 2e^{-16}$ *** |
| Smooth terms:  | edf       | Ref.df     | F      | p-value          |
| $\phi_0(\cdot)$  | 8.641     | 8.963      | 39.175 | $< 2e^{-16}$ *** |
| $\phi_1(\cdot)$  | 5.392     | 6.582      | 1.722  | 0.109            |
| $\phi_2(\cdot)$  | 8.595     | 8.95       | 5.644  | $< 2e^{-16}$ *** |
| $\phi_3(\cdot)$  | 1.235     | 1.434      | 80     | $< 2e^{-16}$ *** |
| R-sq.(adj)=0.752; Dev.expl.=77.5%                                      |           |            |        |                  |
| GCV=6.0497e <sup>-05</sup> ; Scale est.=5.4645e <sup>-05</sup> ; n=257 |           |            |        |                  |

# Distribution Dynamics

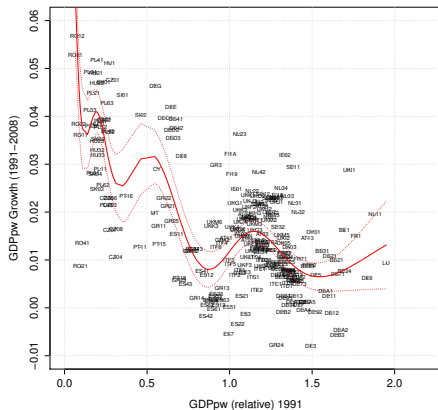


Figura: Absolute convergence in the GDP per worker of 256 European regions. Parametric and nonparametric regression.

## Distribution Dynamics (cont.d)

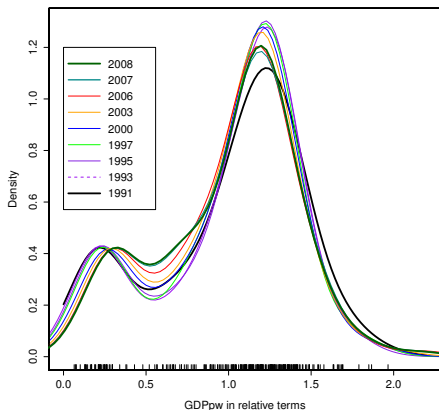


Figura: Estimated distributions of (relative) GDP per worker from 1991 to 2008 in 254 NUTS-2 European regions.