Quantitative Economics for the Evaluation of the European Policy

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Solow model with public expenditure

Consider a Cobb-Douglas production function, and assume that GDP of region i at time t, Y_{it}, is given by:

$$Y_{it} = \Psi(SCF_i) K_{it}^{\alpha} \left(A_{it} L_{it}\right)^{1-\alpha} \tag{1}$$

- SCF_i indicates the amount of structural funds per unit of output;
- $\Psi(\cdot)$ describes their impact on Y_{it};
- the availability of structural funds is not essential to carry out production ($\Psi(0) = 1$);

Limited technological spillovers

$$A_{it} = \Omega_{it} \Pi_{j \neq i}^{N} A_{jt}^{\theta w_{ij}} \tag{2}$$

where:

- Ω_{it} is the part of technological progress of region i not depending on the other regions;
- A_{jt} is the technological progress of region j;
- θ is the parameter measuring the **intensity** of spatial spillovers ;
- w_{ij} the element (i, j) of **spatial matrix** W;
- $\Pi_{j \neq i}^{N} A_{jt}^{\theta w_{ij}}$ is the spatial technological externalities (θ reflects the degree of spatial externalities).

Empirical model

This model yields to the following conditional convergence equation:

$$\gamma = \phi_0 + \mathbf{X}\phi_X + \mathbf{WX}\phi_{WX} + \theta\mathbf{W}\gamma + \mathbf{e}, \tag{3}$$

where:

- γ is the vector of average growth rates of GDP per worker;
- $\mathbf{X}$ is the $(N \times K_X)$ matrix of explanatory variables, i.e. the Solow regressors (investment rates, the augmented employment growth rates and the initial level of GDP per worker) and structural funds SCF;
- $\mathbf{WX}$ is the matrix of spatially lagged exogenous variables;
- $\mathbf{W}\gamma$ is the endogenous spatial lag variable, i.e. the growth rate in neighbouring regions.

Spatial Durbin model

$$\begin{aligned}
\gamma &= \phi_0 + \theta \mathbf{W}\gamma + \mathbf{X}\phi_X + \mathbf{WX}\phi_{WX} + \mathbf{e} \\
&\quad \text{or,} \qquad (4)\\
\gamma &= (\mathbf{I} - \theta\mathbf{W})^{-1}[\mathbf{X}\phi_X + \mathbf{WX}\phi_{WX}] + (\mathbf{I} - \theta\mathbf{W})^{-1}\mathbf{e}
\end{aligned}$$

- *Spatial multiplier effect of global interaction effect*: the outcome in a location i will not only be affected by the exogenous characteristics of i, but also by those in all other locations through the inverse spatial transformation $(\mathbf{I} - \theta\mathbf{W})^{-1}$
- *Spatial diffusion of random shocks*: a random shock in a location i does not only affect the outcome of i , but also has an impact on the outcome in all other locations through the same inverse spatial transformation

Interpreting parameter estimates: direct and indirect effects

In linear regression models:

- *Direct effect* of a change in X_{ik} on region i:

$$\frac{\partial E[\gamma_i]}{\partial X_{ik}} = \phi_X(k) \tag{5}$$

- *Indirect effect* of a change in X_{jk} on region i:

$$\frac{\partial E[\gamma_i]}{\partial X_{jk}} = 0 \tag{6}$$

Direct and indirect effects in SDM

- *Direct effect* of a change in X_{ik} on region i:

$$\frac{\partial E[\gamma_i]}{\partial X_{ik}} = (I - \theta W)^{-1}_{ii}[I\phi_X(k) - W\phi_{WX}(k)] \neq \phi_X(k) \qquad (7)$$

$\Rightarrow$ It includes the effect of feedback loops where observation i affects observation j and observation j also affects i. Its magnitude depends upon: 1) the position of the regions in space, 2) the degree of connectivity among regions which is governed by $\mathbf{W}$, 3) the parameters $\phi_X(k)$, $\phi_{WX}(k)$ and θ.

If one region receives more funds, what will be the expected impact on the growth rate of GDP per worker in that region?

Direct and indirect effects in SDM

- *Indirect effect* of a change in X_{jk} on region i:

$$\frac{\partial E[\gamma_i]}{\partial X_{jk}} = (I - \theta W)^{-1}_{ij}(I\phi_X(k) - W\phi_{WX}(k) \neq 0 \tag{8}$$

  If region j receives more funds, what will be the expected impact on the growth rate of GDP per worker in the region i not receiving any additional fund?

- *Total effect* of a change in X_k on region i:

$$\frac{\partial E[\gamma_i]}{\partial X_k} = \frac{\partial E[\gamma_i]}{\partial X_{ik}} + \frac{\partial E[\gamma_i]}{\partial X_{jk}} \tag{9}$$

  If regions i and j receive more funds, what will be the expected impact on growth rates of GDP per worker of region i?

Estimation technique

In Spatial Durbin Model with exogenous regressors:

- OLS is biased and inconsistent due to the endogeneity $\mathbf{W}\gamma$
- Maximum Likelihood estimation
- Instrumental Variable estimation

Some notes on Maximum Likelihood estimation

- What is the **probability** of observing the data $\{y_i\}$ $i = 1, ..., N$ as a function of the parameters of the model $y = X\beta + \epsilon$?
- In order to compute this probability we need the joint density function for $\{y_i\}$ $i = 1, ..., N$ or, equivalently, for $\{\epsilon_i\}$ $i = 1, ..., N$
- This joint density function is called **likelihood function** (L), for which is necessary to assume the shape of the density function of $\{\epsilon_i\}$ $i = 1, ..., N$.
- $\{\epsilon_i\} \sim N(0, \sigma^2)$; then, for each i:

$$f(\epsilon_i) = \frac{1}{\sqrt{\sigma^2(2\pi)}} e^{-\frac{\epsilon^2}{2\sigma^2}} \tag{10}$$

Some notes on Maximum Likelihood estimation

- Assuming that the N observations are iid $\sim N(0, \sigma^2)$, the **joint density function** is:

$$\begin{aligned} L(\beta, \sigma^2) &= f(y_1|X\beta, \sigma^2)...f(y_N|X\beta, \sigma^2) \\ &= f(\epsilon_1)...f(\epsilon_N) \\ &= \prod_{i=1}^{N} f(\epsilon_i) = \prod_{i=1}^{N} \frac{1}{\sqrt{\sigma^2(2\pi)}} e^{-\frac{\epsilon^2}{2\sigma^2}} \end{aligned}$$

- In matrix form:

$$L(\beta, \sigma^2) = \prod_{i=1}^{N} \frac{1}{\sigma^2(2\pi)^{N/2}} e^{-\frac{\epsilon' \epsilon}{2\sigma^2}}$$

Some notes on Maximum Likelihood estimation

- Taking the logs:

$$\begin{aligned} lnL(\beta, \sigma^2) &= -\frac{N}{2} ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}(\epsilon'\epsilon) \\ &= -\frac{N}{2} ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}(y'y - 2\beta'X'y - \beta'X'X\beta) \end{aligned}$$

- Which values of β and σ^2 maximize the likelihood function?

$$\begin{aligned} \hat{\beta}_{ML} &= (X'X)^{-1}X'y \\ \hat{\sigma}^2_{ML} &= RSS/N \end{aligned}$$

- Moreover:

$$[I(\Theta)]^{-1} = \left(-E\left[\frac{\partial^2 lnL}{\partial\Theta\partial\Theta'}\right]\right)^{-1}$$

where $\Theta = \{\beta, \sigma^2\}$.

ML for SDM

When the model is a SDM as:

$$\begin{aligned} y &= \theta W y + X\beta + WX\delta + \epsilon \;\; \epsilon \sim N(0, \sigma^2 I_N) \\ \epsilon &= y - \theta W y - X\beta - WX\delta \end{aligned}$$

Therefore:

$$\begin{aligned} lnL(y|\theta, \beta, \delta, \sigma^2) = -\frac{N}{2} ln(2\pi\sigma^2) - ln|I_N - \theta W| + \\ - \frac{1}{2\sigma^2}\left[(y - \theta W y - X\beta - WX\delta)'(y - \theta W y - X\beta - WX\delta)\right] \end{aligned}$$

R package

spdep

lagsarlm, impacts