Quantitative Economics for the Evaluation of the European Policy

Dipartimento di Economia e Management

Co-funded by the Erasmus+ Programme of the European Union

Project funded by
European Commission Erasmus + Programme –Jean Monnet Action
Project number 553280-EPP-1-2015-1-IT-EPPJMO-MODULE

Irene Brunetti Davide Fiaschi Angela Parenti[1]

20/11/2015

[1] ireneb@ec.unipi.it, davide.fiaschi@unipi.it, and aparenti@ec.unipi.it.

Solow model with public expenditure

Consider a Cobb-Douglas production function, and assume that GDP of region i at time t, Y_{it}, is given by:

$$Y_{it} = \Psi(SCF_i) K_{it}^{\alpha} \left(A_{it} L_{it}\right)^{1-\alpha} \tag{1}$$

- SCF_i indicates the amount of structural funds per unit of output;
- $\Psi(\cdot)$ describes their impact on Y_{it};
- the availability of structural funds is not essential to carry out production ($\Psi(0) = 1$);

Limited technological spillovers

$$A_{it} = \Omega_{it} \Pi_{j \neq i}^{N} A_{jt}^{\theta w_{ij}} \tag{2}$$

where:

- Ω_{it} is the part of technological progress of region i not depending on the other regions;
- A_{jt} is the technological progress of region j;
- θ is the parameter measuring the **intensity** of spatial spillovers ;
- w_{ij} the element (i, j) of **spatial matrix** W;
- $\Pi_{j \neq i}^{N} A_{jt}^{\theta w_{ij}}$ is the spatial technological externalities (θ reflects the degree of spatial externalities).

Empirical model

This model yields to the following conditional convergence equation:

$$\gamma = \phi_0 + \mathbf{X}\phi_X + \mathbf{WX}\phi_{WX} + \theta\mathbf{W}\gamma + \mathbf{e}, \tag{3}$$

where:

- γ is the vector of average growth rates of GDP per worker;
- $\mathbf{X}$ is the $(N \times K_X)$ matrix of explanatory variables, i.e. the Solow regressors (investment rates, the augmented employment growth rates and the initial level of GDP per worker) and structural funds SCF;
- $\mathbf{WX}$ is the matrix of spatially lagged exogenous variables;
- $\mathbf{W}\gamma$ is the endogenous spatial lag variable, i.e. the growth rate in neighbouring regions.

Point of departure: classical linear regression model

The classical regression model is:

$$\mathbf{y} = \mathbf{X}\beta + \mathbf{u} \quad E[\mathbf{u}] = 0;\ E[\mathbf{u}\mathbf{u}'] = \sigma^2\mathbf{I} \tag{4}$$

where:

- No spatial diffusion and/or no spatial interaction effects

  $\Rightarrow \frac{\partial E[y_i]}{\partial X_{ik}} = \beta_k$ and $\frac{\partial E[y_j]}{\partial X_{jk}} = 0$

$\Rightarrow$ the **spatial dependence** is neglected!

Source of spatial dependence

- **Spatial spillovers**: technological interdependence between regions, factor movement, spillover effect
- **Omitted spatially correlated variables**: unobserved factors as social capital or local amenities
- **Measurement errors and unobserved heterogeneity**: administrative boundaries that do not reflect the economic phenomenon under study (e.g., reallocation of SCF)

Consequences of spatial dependence

- The presence of spatial dependence violates one of the assumptions of the classical regression model: independence
  - This creates a problem in assessing statistical inference: the errors in the regression model can no longer be assumed to have zero covariances with each other
- The presence of spatial dependence in the dependent variable implies a simultaneous model
  - Endogeneity of $\mathbf{W}\gamma$ implying the inconsistency of OLS estimator.

A taxonomy of spatial models

- **Spatial cross-regressive model**: $\mathbf{y} = \mathbf{X}\beta + \mathbf{WX}\delta + \mathbf{u}$
  $\Rightarrow$ *Local externalities*: if **X** is exogenous, also **WX** is exogenous and the model can be estimated by OLS
- **Spatial error model**: $\mathbf{y} = \mathbf{X}\beta + \epsilon \quad \epsilon = \lambda\mathbf{W}\epsilon + \mathbf{u}$
  $\Rightarrow$ *Global spatial diffusion in random shocks*: the structure of its variance-covariance matrix is such that every location is correlated with every other location in the system, but closest locations more so: heteroskedasticity

A taxonomy of spatial models

- **Spatial lag or Spatial Autoregressive model**: $\mathbf{y} = \rho \mathbf{W}\mathbf{y} + \mathbf{X}\beta + \mathbf{u}$
  $\Rightarrow$ *Spatial multiplier effect of global interaction effect*: the outcome in a location *i* will not only be affected by the exogenous characteristics of *i*, but also by those in all other locations through the inverse spatial transformation $(I - \rho W)^{-1}$:

$$E[\mathbf{y}|\mathbf{X}] = \mathbf{X}\beta + \rho \mathbf{W}\mathbf{X}\beta + \rho^2 \mathbf{W}^2 \mathbf{X}\beta + ... \tag{5}$$

  $\Rightarrow$ *Spatial diffusion of random shocks*: a random shock in a location *i* does not only affect the outcome of *i* , but also has an impact on the outcome in all other locations through the same inverse spatial transformation

- **Spatial Durbin model**: $\mathbf{y} = \rho \mathbf{W}\mathbf{y} + \mathbf{X}\beta + \mathbf{W}\mathbf{X}\delta + \mathbf{u}$

Interpreting parameter estimates: direct and indirect effects

- In SAR and SDM, a change in a single observation (region) associated with any given explanatory variable will affect the region itself (a direct impact) and potentially affect all other regions indirectly (an indirect effect) thourgh the spatial multiplier mechanism
- In linear regression models:

$$\frac{\partial E[y_i]}{\partial X_{ik}} = \beta_k \quad \frac{\partial E[y_i]}{\partial X_{jk}} = 0 \tag{6}$$

Direct and indirect effects in SDM

- *Direct effect* of a change in X_{ik} on region i:

$$\frac{\partial E[y_i]}{\partial X_{ik}} = (I - \rho W_{ii})^{-1}(I\beta_k - W\delta_k) \neq \beta_k \tag{7}$$

  $\Rightarrow$ It includes the effect of feedback loops where observation i affects observation j and observation j also affects i. Its magnitude depends upon: 1) the position of the regions in space, 2) the degree of connectivity among regions which is governed by **W**, 3) the parameters β_k, δ_k and ρ.

- *Indirect effect* of a change in X_{jk} on region i:

$$\frac{\partial E[y_i]}{\partial X_{jk}} = (I - \rho W_{jj})^{-1}(I\beta_k - W\delta_k) \neq 0 \tag{8}$$