Quantitative Economics for the Evaluation of the European Policy

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Alternative methodology for growth accounting

Solow (1957) proposes a methodology to calculate the contribution of individual factors to growth.

Two crucial assumptions:

- Production function is assumed to be to constant returns to scale (e.g. Cobb-Douglas $Y = K^{\alpha}(AH)^{1-\alpha}$).
- Factor are paid to their marginal productivities, i.e. $r = \partial Y / \partial K$ and $W/P = \partial Y / \partial L$ with $H = hL$.

The two assumptions are related, because if we assume that factors are paid to their marginal productivities then we have to assume constant returns to scale to order to have a complete distribution of output. Indeed:

$$rK + \frac{W}{P} L = \frac{\partial Y}{\partial K} K + \frac{\partial Y}{\partial L} L = Y = F(K, AH) \tag{1}$$

Constant return to scale technology

A very general production function is represented by CES (constant elasticity of substitution) technology, i.e.

$$Y = \left[\alpha K^{\rho} + (1-\alpha)(AH)^{\rho}\right]^{1/\rho}, \tag{2}$$

where $1/(1+\rho)$ measures the elasticity of substitution between K and H. The most important cases:

- $\rho = 0$, i.e. an elasticity equal to one, corresponds to Cobb-Douglas ($Y = K^{\alpha}(AH)^{1-\alpha}$);
- $\rho = -1$ corresponds to the case of perfect substitution between factors ($Y = \alpha K + (1-\alpha) H$); and
- $\rho \to \infty$ to the case of Leontief technology ($Y = \min\{\alpha K, (1-\alpha) AH\}$).

Consider the Cobb-Douglas technology:

$$Y = K^{\alpha} (AH)^{1-\alpha}; \tag{3}$$

take the logarithms of both sides and the first derivative with respect to time, i.e.

$$\log Y = \alpha \log K + (1-\alpha) \log H + (1-\alpha) \log A \tag{4}$$

and

$$\frac{\dot{Y}}{Y} = g_Y = \alpha g_K + (1-\alpha) g_H + (1-\alpha) g_A \tag{5}$$

We can observe that g_Y, g_K and $g_H = g_h + g_L$ are observable but not α and g_A.

However, from the assumption of competitive factor markets we have that:

$$\frac{rK}{Y} = \frac{\partial Y / \partial K r}{Y} = \alpha; \tag{6}$$

this means that a measure of α is the share of profits on total output.

Therefore:

$$g_A = \frac{g_Y - \alpha g_K - (1-\alpha)(g_h + g_L)}{1-\alpha} \tag{7}$$

The application of the procedure to our sample leads to an estimate of the distribution of the growth rate of g_A, also denoted Total Factor Productivity, with the following characteristics:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-0.051490	-0.020280	-0.012840	-0.010950	-0.003239	0.030250

One of the most important drawback of this methodology is that any omitted factor in the production function (e.g. natural resources) is attributed to the growth rate of technological progress. This is the reason because g_A is also denoted as *Solow residual*.

Solow model with public expenditure

Barro (1990) proposes a model where public expenditure has a positive impact on the productivities of private factors. In particular:

$$Y = K^{\alpha} H^{1-\alpha} G^{1-\alpha}, \tag{8}$$

where G is the total amount of public expenditure.
Assuming that public expenditure is financed in balanced budget with a flat tax rate on income:

$$G = \tau Y, \tag{9}$$

where τ is the tax rate, then:

$$Y = K^{\alpha} H^{1-\alpha} (\tau Y)^{1-\alpha}, \tag{10}$$

i.e.

$$Y = KH^{(1-\alpha)/\alpha} \tau^{(1-\alpha)/\alpha} \tag{11}$$

The net income of economy is given by:

$$(1-\tau)\,Y = (1-\tau)\,KH^{(1-\alpha)/\alpha}\tau^{(1-\alpha)/\alpha}; \tag{12}$$

the maximum net income is reached for $\tau = 1-\alpha$.
This result is mainly due to the specification of production function.

To complete the model we can add an equation for the accumulation of capital.